CHAPTER 9

RELIABILITY GROWTH

INTRODUCTION

Initial prototype models of complex weapon systems will invariably have inherent reliability and performance deficiencies that generally could not have been foreseen and eliminated in early design stages. To uncover and eliminate these deficiencies, we subject these early prototypes and later more mature models to a series of development and operational tests. These tests have been specifically planned to stress the system components to predetermined realistic levels at which inadequate design features will surface as system failures. These failures are analyzed, design modifications incorporated, and then the modified system is tested to verify the validity of the design change.

This testing philosophy utilizes the test-analyze-fix-test (TAFT) procedure as the basic catalyst in achieving system reliability growth. The ultimate goal of a reliability growth program, and, ideed, the entire test program, is to increase system reliability to stated requirement levels by eliminating a sufficient number of inherent system failure modes.

A successful system reliability growth program is dependent on several factors. First, an accurate determination must be made of the current system reliability status. Second, a test program must be planned which subjects the system to test exposure and stress levels adequate to uncover inherent failure models and to verify design modifications. Third, the program manager must address the availability of test schedule and resource required to support the "TAFT" procedure.

To adequately control the above and other factors inherent in the reliability growth process, it is important to track reliability growth throughout the testing program. This is accomplished by periodically assessing system reliability (e.g., at the end of every test phase) and comparing the current reliability to the planned level of achievement for that point in time. These assessments provide the necessary data and visibility to support necessary corrective management initiatives.

The following paragraphs present the analytical tools required to plan a reliability growth program and those useful in tracking the actual growth of a system during consecutive test phases.

RELIABILITY GROWTH CONCEPTS

Idealized Growth

For a system under development, reliability generally increases rapidly early on and at a much slower rate towards the end of development. It is useful at the beginning of a development program to depict the growth in reliability as a smooth curve which rises at slower and slower rates as time progresses. This curve, known as the idealized growth curve, does not necessarily convey precisely how the reliability will actually grow during development. Its purpose is to present a preliminary view as to how a program should be progressing in order for the final reliability requirements to be realized. The model for the idealized curve is the Duane Growth Model, the primary feature of which is the every decreasing rate of growth as testing progresses.

The development testing program will usually consist of several major test phases. Within each test phase, the testing may be conducted according to a program which incorporates fixes or design changes while testing is in process, at the end of the test phase, or both. If we divide the development testing program into its major phases and join by a smooth curve the proposed reliability values for the system at the end of these test phases , the resulting curve represents the overall pattern for reliability growth. This is called the idealized reliability growth curve. The idealized curve is very useful in quantifying the overall development effort and serves as a significant tool in the planning of reliability growth.

Planned Growth

The planning of reliability growth is accomplished early in the development program, before hard reliability data are obtained, and is typically a joint effort between the program manager and the contractor. Its purpose is to give a realistic and detailed indication of how system reliability enhancement is planned to grow during development. Reliability growth planning addresses program schedules, testing resources and the test exposure levels. The objective of growth planning is to determine the number and length of distinct test phases, whether design modifications will be incorporated during or between distinct test phases and the increases in reliability to ensure that the achieved reliability remains within sight of the idealized growth values.

Growth Tracking

The primary objective in tracking reliability growth is to obtain demonstrated reliability values at the end of each test phase. The demonstrated reliability is usually determined by one of two methods. The first and preferred method is reliability growth analysis. However, should the data not lend themselves to this type of analysis, then the second method, an engineering analysis, should be used. Reliability growth analysis is useful for combining test data to obtain a demonstrated estimate in the presence of changing configurations within a given test phase. Engineering analysis is employed when the reliability growth analysis procedure is inappropriate. We do not address engineering analysis in this text.

IDEALIZED GROWTH CURVE DEVELOPMENT

The first step in planning reliability growth is the development of an idealized growth curve. The development of this curve is based on the following three parameters :

t_1 = length of initial test phase.

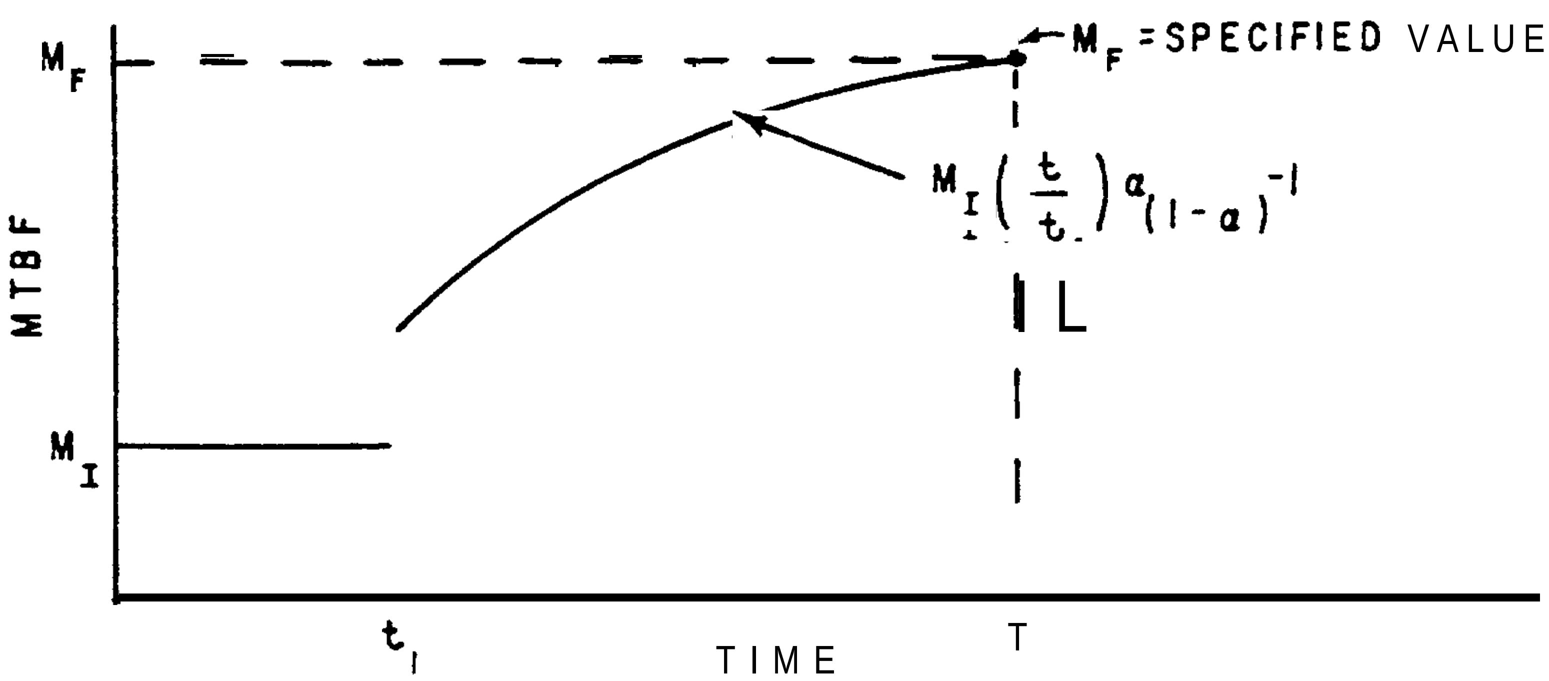
M_I = average MTBF over the first test phase, t_1.

α = a parameter which addresses the rate of growth.

The idealized curve, illustrated in Figure 9-1, is a graph of the function M(t) where:

$$M(t) = \begin{cases} M_I & \text{in the interval } 0<t\leq t_1 \\ & \text{and} \\ M_I(t/t_1)^{\alpha}(1-\alpha)^{-1} & \text{in the interval } t>t_1 \,. \end{cases} \tag{9.1}$$

FIGURE 9-1 IDEALIZED GROWTH CURVE



The idealized growth curve development procedure starts with the determination of the initial test phase length (t_1) and the average MTBF over the initial test phase (MI) . There is no exact procedure for determining values of these parameters. The initial test phase length (t_1) may be determined through a joint effort of both the contractor and the program manager. Perhaps an initial test has already been performed, in which case both t_1 and M_I are known. If this is not the case, then the determination of a value for M_I would in all likelihood require the expertise of individuals familiar with present day capabilities of the actual system in question or other similar systems. The parameter, M_I , should be a realistic estimate of what the system's average MTBF will be during the initial test phase, i.e. , before any significant design weaknesses can be detected and modifications developed, implemented and tested.

The parameter α represents the rate of growth necessary to achieve an MTBF of M_F (the contractually specified value) after a total of T hours of testing. The specified value M_F represents the user's desired capability and is determined by means of extensive battlefield as well as logistics analyses. The total amount of testing T is a value which is determined through a joint contractor and program manager effort and is based upon considerations of calendar time and number of prototypes available in addition to cost constraints. For fixed values of t_1, M_I, T, and M_F, the value for α is calculated algebraically by solving the equation

$$M_F = M_I(T/t_1)^{\alpha}(1-\alpha)^{-1} . \qquad (9.2)$$

There is no closed form solution for α in equation 9.2. However, an approximation for α is given below.

$$\alpha = \log_e(t_1/T)-1 + \{(\log_e(T/t_1)+1)^2 + 2\log_e(M_F/M_I)\}^{1/2} . \qquad (9.3)$$

This is a reasonably good approximation when α is smaller than 0.4. The approximation will always be on the high side but within two decimal places for values of α less than 0.3. Programs which require a growth rate (α) greater than 0.3 should be viewed somewhat skeptically and those which require an α greater than 0.4 are far too ambitious to be realistic.

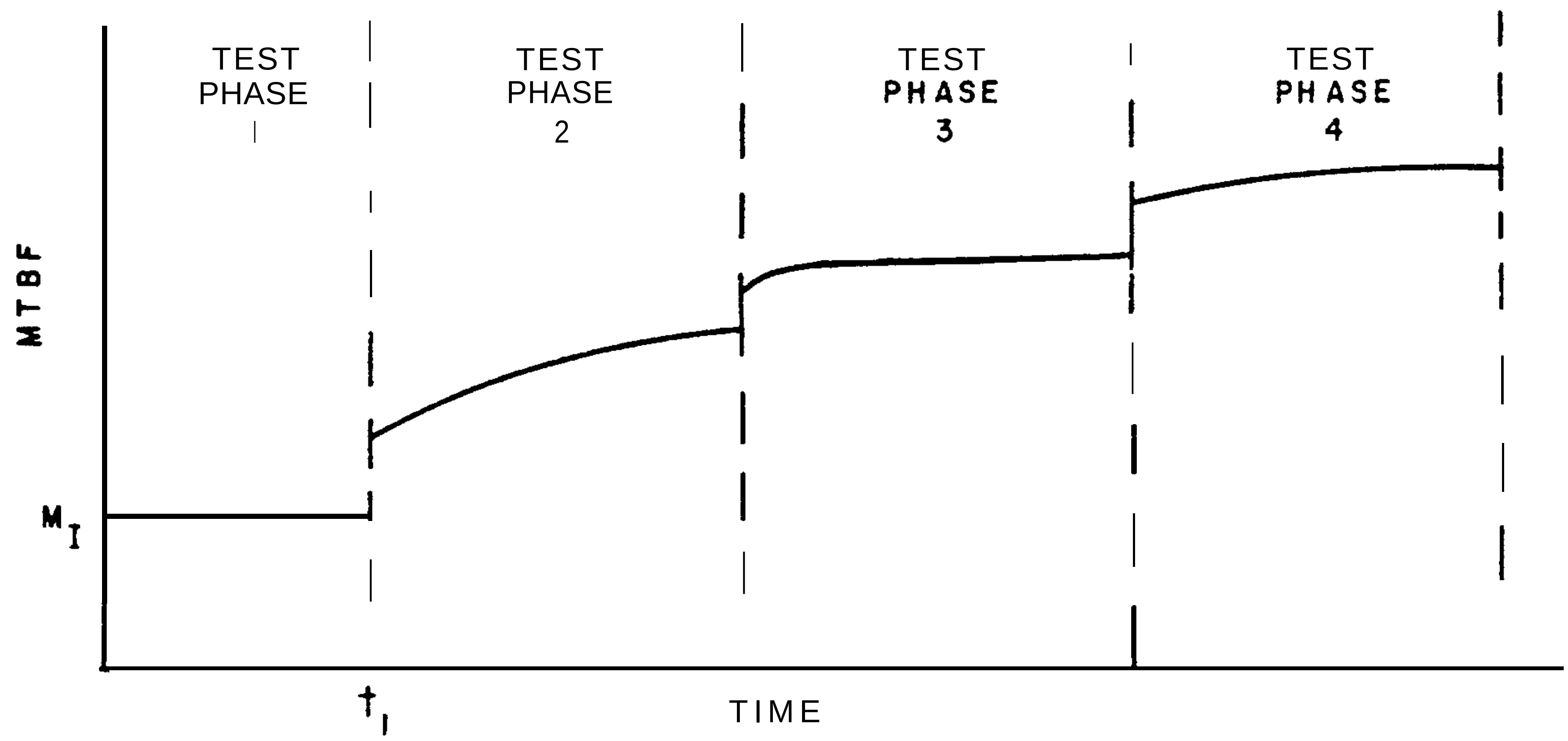
PLANNED GROWTH CURVE DEVELOPMENT

Once the idealized curve has been constructed, it is used as a basis for developing a planned growth curve. The planned growth curve displays, in graphic terms, how the producer plans by stages to achieve the required final MTBF. The curve is divided into portions which represent the different test phases. The entire curve indicates graphically where in the development program reliability is expected to grow, and where it is expected to remain constant. The curve depicts increases in reliability resulting from design improvements. At any given time during development testing, the planned growth curve value can be higher than, lower than, or equal to the idealized growth curve value. The idealized curve serves as a guide for the preparation of the planned curve. At no time in the planning of reliability growth should the separation between values on the curve be large. If this is the case, then unquestionably there is some point during development where an unrealistic jump in reliability is expected to occur.

As we mentioned earlier, the planned growth curve should graphically display how reliability is expected to grow. Growth, of course, will generally occur as a result of incorporating design modifications. These modifications may be incorporated during the test phase, resulting in a smooth gradual improvement in reliability, or at the end of the test phase, resulting in a jump in reliability from the end of one test phase to the beginning of the subsequent test phase. In Figure 9-2, we present a planned growth curve which illustrates the effect on reliability of design improvements incorporated during, and at the completion of, the various test phases. Note that the rate of growth is gradually decreasing as the system matures.

The portion of the planned growth curve between time zero and t_1 is identical to the idealized growth curve.

FIGURE 9-2 PLANNED GROWTH CURVE

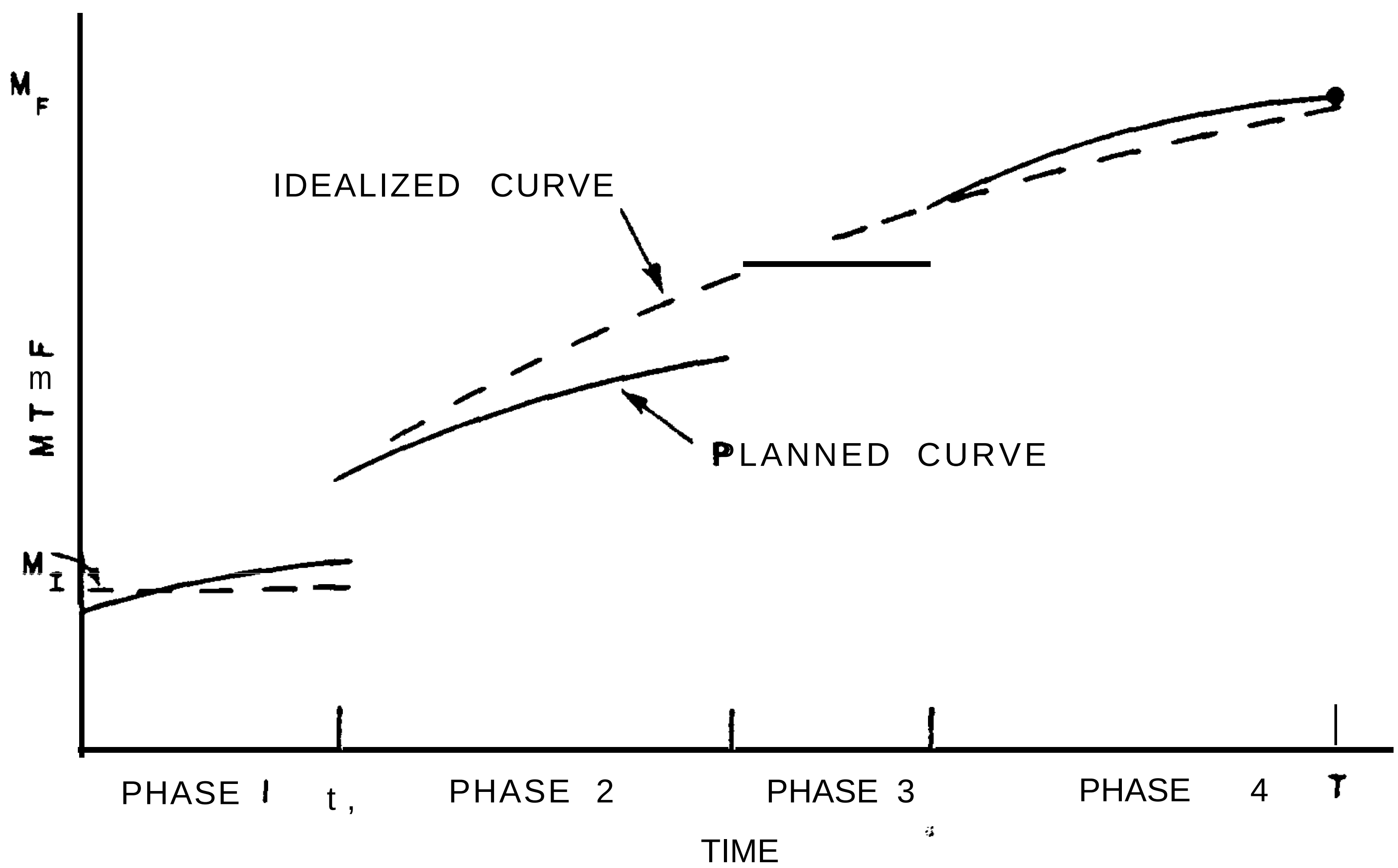


Delayed fixes are incorporated after each of the first three test phases. During all of test phase 2 and early in test phase 3, fixes are incorporated. Fixes are incorporated during the final test phase, and the MTBF grows to the required specified value. It is not a good practice to allow for a jump in reliability at the end of the final test phase even though fixes may be incorporated. The reason is that there is no test time available to determine the impact of these fixes.

The planned growth curve is an indication of how the required MTBF might be achieved and is developed by using the idealized curve as a guide.

Figure 9-3 illustrates the graphical relationship between the planned growth curve and the corresponding idealized curve. A point on the planned curve at any given time in the program represents the level of reliability to be achieved at that time.

FIGURE 9-3 EXAMPLE OF A PLANNED GROWTH CURVE AND CORRESPONDING IDEALIZED CURVE



RELIABILITY GROWTH TRACKING

The objectives of growth tracking include:

- Determining if growth is occurring and to what degree,
- Estimating the present reliability, and
- Formulating a projection of the reliability expected at some future time.

The methods discussed in this section are directed toward reliability growth tracking using a mathematical model. Parameters of the model are estimated using data which have been accumulated during a given test phase. Using this model and the parameter estimates, we can determine present and projected reliability values. The present value represents the reliability inherent in the existing configuration. A projected value represents the reliability of the system expected at some future time. Projected values take into account the effect of design improvements intended to correct observed failure modes or failure modes which further testing will surface. Generally growth tracking analysis is performed at the end of a major test phase.

The mathematical model we shall use for growth tracking describes the failure rate as a function of time. The value r(t) denotes the failure rate of the system after t units of testing, and

$$r(t) = \lambda\beta t^{\beta-1}, \tag{9.4}$$

where A and β are parameters of the model which determine the scale and the shape of the curve. The reciprocal of r(t) is the MTBF of the system after t units of testing. We fit the model to the actual test data using maximum likelihood estimates for A and β. (See Chapter 6 for a discussion of maximum likelihood estimates.)

When actual failure times $(t_1, t_2, \ldots t_N)$ are known and the test phase is <u>time truncated,</u> i.e. , at time T, the estimate for β is

$$\hat{\beta} = \frac{N}{N\log_e T - \sum_{i=1}^{N} \log_e t_i} \tag{9.5}$$

The estimate for A is

$$\hat{\lambda} = N/T^{\hat{\beta}}. \tag{9.6}$$

When the test phase is <u>failure truncated,</u> i.e. , at time t_N, the estimates are

$$\hat{\beta} = \frac{N}{(N-1)\log_e t_N - \sum_{i=1}^{N-1} \log_e t_i}, \tag{9.7}$$

and

$$\hat{\lambda} = N/T^{\hat{\beta}}. \tag{9.8}$$

In either case, the estimate of r(t) is

$$\hat{r}(t) = \hat{\lambda}\hat{\beta}t^{\hat{\beta}-1}. \tag{9.9}$$

The reciprocal of $\hat{r}(t)$ is the estimate of the MTBF of the system after a test period of length t, that is

$$\hat{M}(t) = \frac{1}{\hat{r}(t)}.$$

Confidence limits for MTBF may be determined by multiplying point estimates of MTBF by the multipliers found in Table 9 of Appendix B.

When actual failure times are not known, the calculation of maximum likelihood estimates requires a complicated iterative procedure which can only be achieved using a computer algorithm. In addition, the estimates are not as accurate as they would be if actual failure times are known and used. It is important then to collect the actual failure times (in total test time) during development testing. See Chapter 10 for more information on this topic.

In Case Studies 9-1 and 9-2, we demonstrate the procedures for preparing idealized and planned growth curves and for tracking reliability growth.

CASE STUDY NO. 9-1

Background

A new helicopter system has been proposed. It is required to have a Mean Time between Mission Failure (MTBMF) of 50 hours. Past experience has shown that an average MTBMF of 20 hours can be expected during the initial test phase. Four test phases are planned, and the manufacturer intends to use test-analyze-fix-test (TAFT) during all but the final test phases. Delayed fixes will be incorporated at the end of all but the final test phase.

Determine

1. Construct the idealized curve for the program when the initial test phase is 100, 200, 300 hours, and the total test time is 1000 hours.

2. Construct an idealized curve and a planned growth curve when the total test time is 2000 hours, and the four test phases are of equal length.

Solutions

la. $t_1 = 100$ $\quad T = 1,000$

$M_I = 20$ $\quad M_F = 50$

i. Solve for α in the model, using the approximation 9.3

$$\alpha = \log_e(100/1000)-1 + \{(\log_e(1000/100)+1)^2$$

$$+ 2\log_e(50/20)\}^{1/2}$$

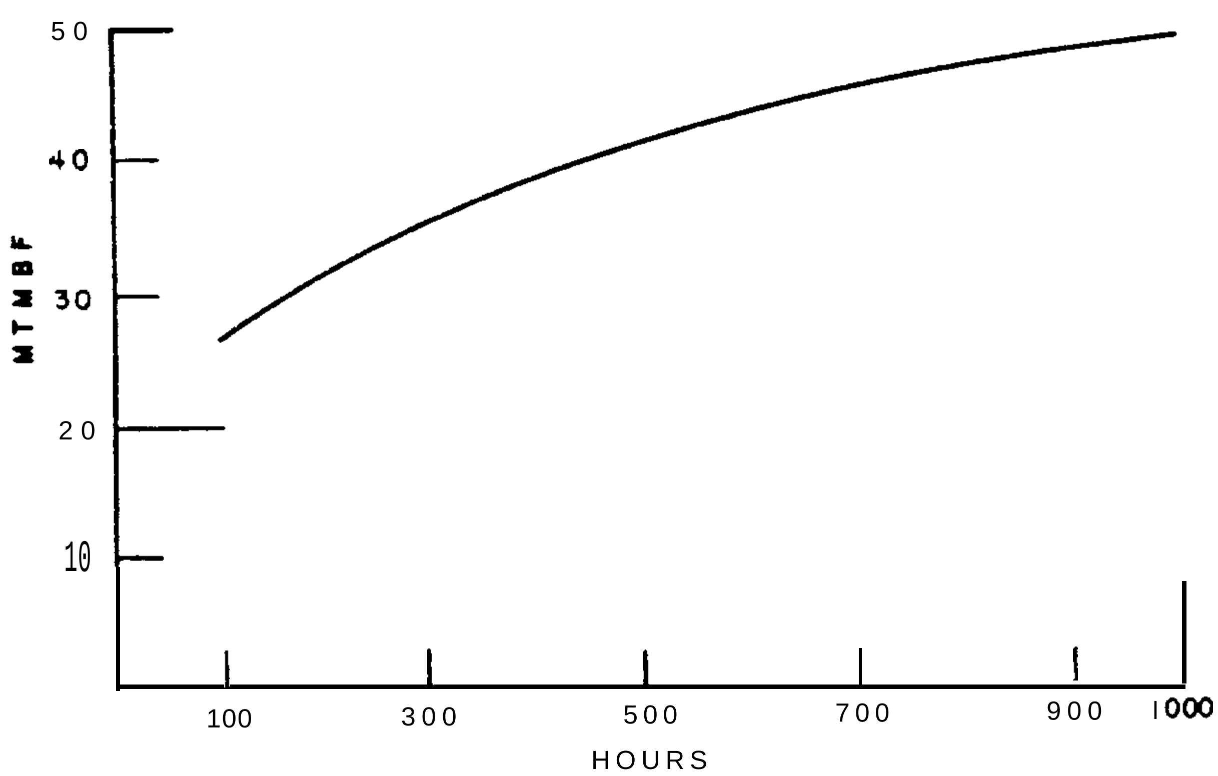
$$= 0.267$$

ii. Determine points on the curve using equation 9.1

$$M(t) = M_I(t/t_1)^{\alpha}(1-\alpha)^{-1}$$

t	M(t)
<100	20
100	27
300	36
500	42
700	46
900	49
1000	50

iii. Sketch the curve.



lb. $t_1 = 200$ $T = 1000$

$M_I = 20$ $M_F = 50$

i. Solve for α

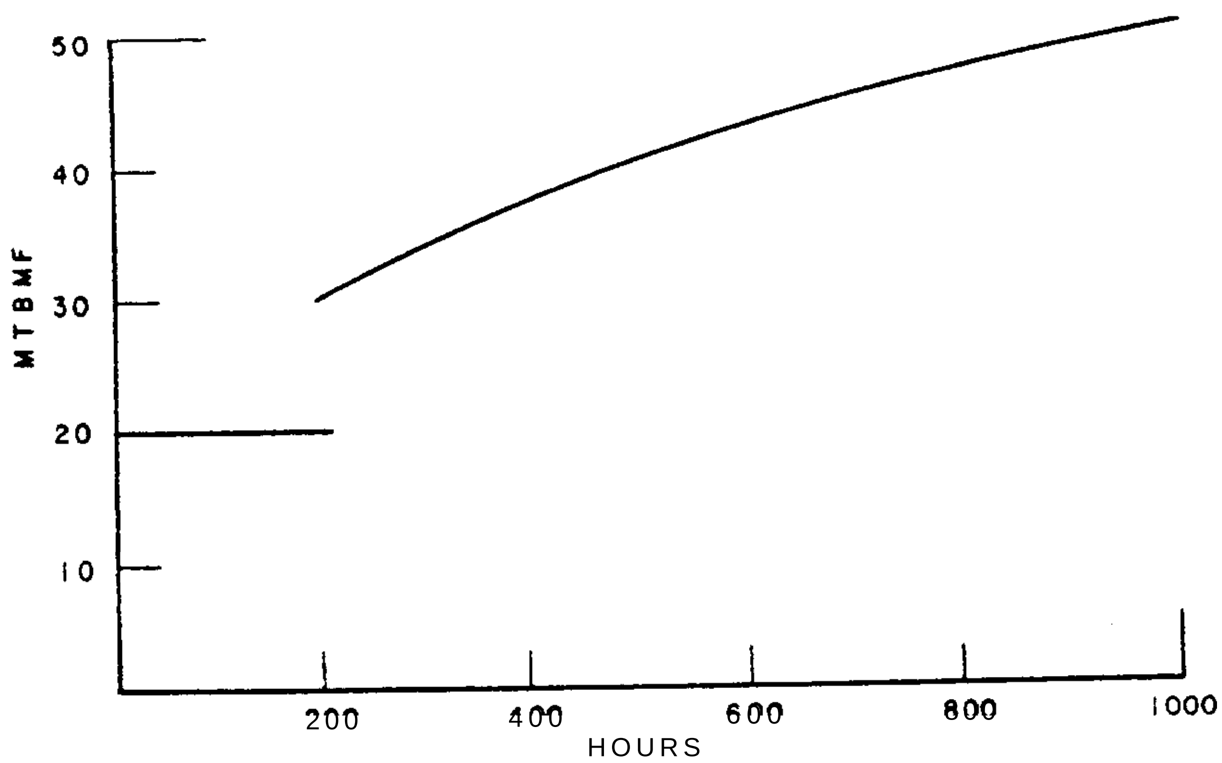
$$\alpha = \log_e(200/1000)-1 + \{(\log_e(1000/200)+1)^2 + 2\log_e(50/20)\}^{1/2}$$

$$= 0.33$$

ii. Determine points on the curve, using equation 9.1

t	M(t)
<200	20
200	30
400	37
600	43
800	47
1000	50

iii. Sketch the curve.



l.c. $t_1 = 300$ $T = 1000$

$M_I = 20$ $M_F = 50$

i. Solve for α

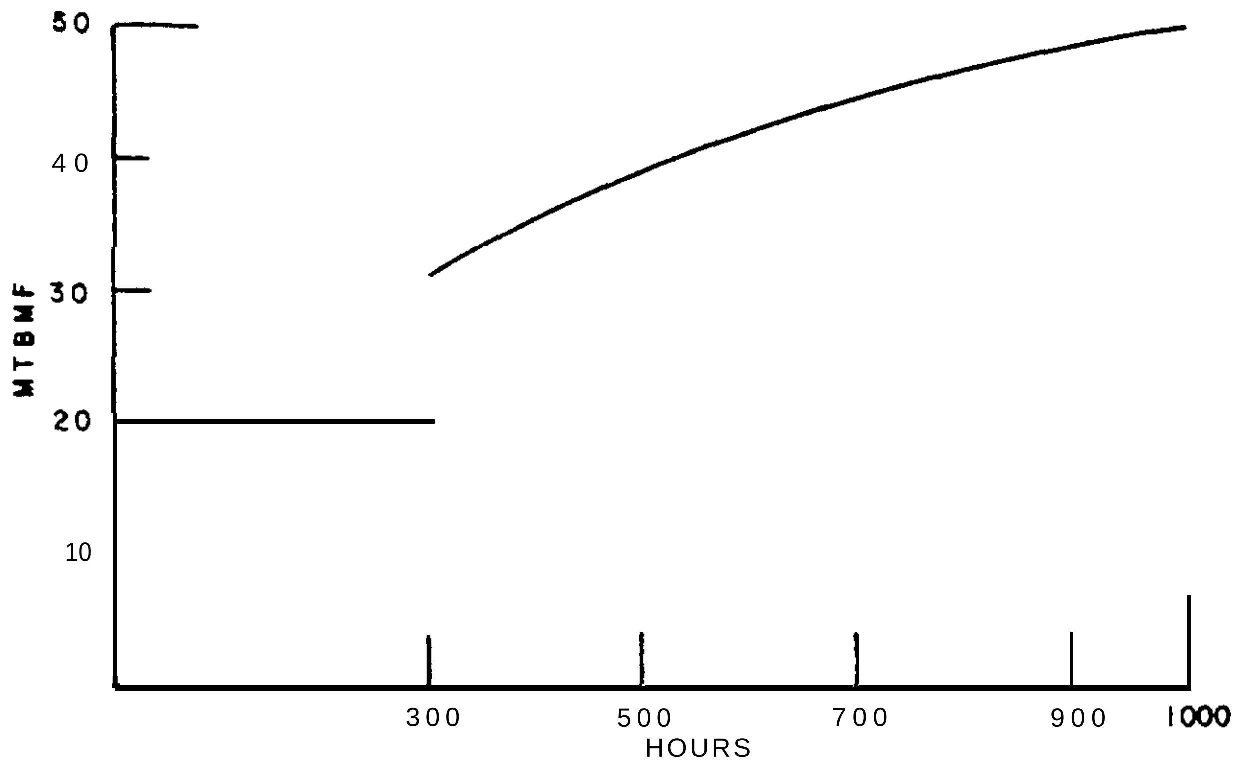
$$\alpha = \log_e(300/1000) - 1 + \{(\log_e(1000/300)+1)^2 + 2\log_e(50/20)\}^{1/2}$$

$$= 0.38$$

ii. Determine points on the curve, using equation 9.1

<u>t</u>	M(t)
<300	20
300	32
500	39
700	44
900	49
1000	50

iii. Sketch the curve



2. $'_1 = 500$ $\quad$ $T = 2000$

$M_I = 20$ $\quad$ $M_F = 50$

i. Solve for α

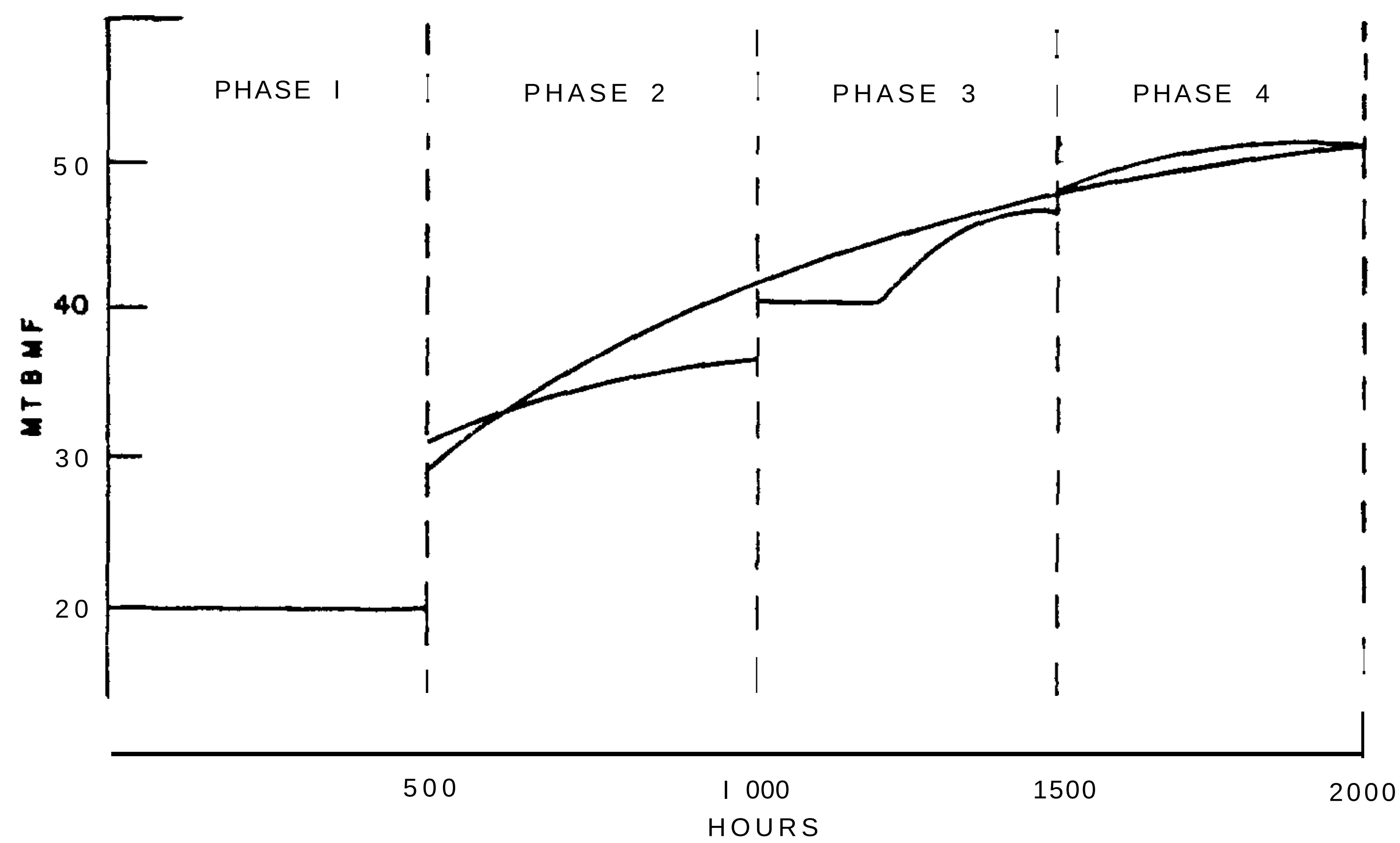
$$\alpha = \log_e(500/2000) - 1 + \{(\log_e(2000/500)+1^2 + 2\log_e(50/20))\}^{1/2}$$

$$= 0.356$$

ii. Determine points on the idealized curve, using equation 9.1

t	M(t)
<500	20
500	31
1000	40
1500	46
2000	50

iii. Sketch the idealized curve and superimpose a planned growth curve.



Commentary

1. Note, for the solution to question 1, how the length of the initial test phase (t_1) affects the growth parameter α. The α of 0.38 in part c. may even be too ambitious. The initial test phase **length** (t_1) should, however, be long enough so that the average MTBF of M_I is achievable.

2. Note that, at various times during the test, the planned growth curve either exceeds or falls below the idealized curve. The relatively low values of the planned curve toward, and at the end of, the second test phase may be cause for some concern. Some fairly substantial increases in reliability are required during the third test phase to get the program back on track.

CASE STUDY NO. 9-2

Background

For the system proposed in Case Study No. 9-1, the data from the final test phase have been collected. The failure times as measured in total test hours are {12, 70, 105, 141, 172, 191, 245, 300, 340, 410, 490}.

Determine

1. Calculate the MTBMF of the system at the end of the final test phase.

2. Calculate a 90% lower limit on the system MTBMF.

Solution

la. If we assume no growth during this test phase, the estimated MTBMF is the ratio of the total time to the number of failures . This value is 500/11 or 45.4 hours.

lb. If fixes are being incorporated during this test phase as suggested in the background for Case Study No. 9-1, then a reliability growth analysis is more appropriate than one based upon the assumption of no growth, as in la.

i. Maximum likelihood estimate for β is

$$\hat{\beta} = 11/\{((11)\ \log_e 500) - (\log_e 12+\log_e 70+\log_e 105+\log_e 141+$$

$$\log_e 172+\log_e 191+\log_e 245+\log_e 300+\log_e 340+\log_e 410+\log_e 490)\}$$

$$\hat{\beta} = 0.89$$

ii. Maximum likelihood estimate for λ is

$$\hat{\lambda} = \frac{11}{(500)^{0.89}} = 0.044$$

iii. Estimated MTBMF after final test phase

$\hat{r}(500) = 0.0198$, and the estimated MTBMF is the reciprocal of $\hat{r}(500)$, which is 50.6 hours.

2. Using Appendix B, Table 9,. for a time terminated test, we find the lower 90% confidence limit multiplier for 11 failures to be 0.565. The lower limit is

$$\theta \geq \theta_L$$

$$\geq (0.565)\hat{\theta}$$

$$\geq (0.565)(50.6)$$

$\geq 28.6.$

We are 90% confident that the true MTBMF is at least 28.6 hours.

Commentary

The estimated MTBMF assuming no growth is 45.4 hours. (See la. above.) Note that this estimate was computed using the number of failures and does not take into account the actual failure times. It cannot show that the times between successive failures seem to be increasing. If fixes are being incorporated during the test, then the reliability growth analysis is more appropriate- With this analysis, the estimated MTBMF is 50.6 hours. The type of analysis used, however, should not be determined by the data but rather by a realistic assessment of the test program.